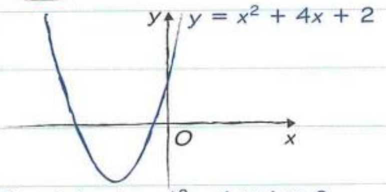
The discriminant

The discriminant of a quadratic expression $ax^2 + bx + c$ is the value $b^2 - 4ac$. You can use the discriminant to work out whether a quadratic equation has any **real roots** or **real solutions**. There are three possible conditions for the discriminant:

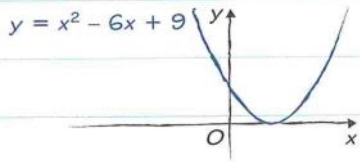
1 $b^2 - 4ac > 0$



Discriminant $= 4^2 - 4 \times 1 \times 2$
$= 8 > 0$

Two distinct real roots

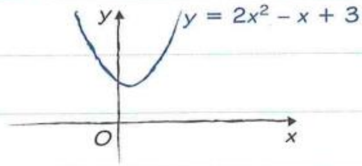
2 $b^2 - 4ac = 0$



Discriminant $= (-6)^2 - 4 \times 1 \times 9$
$= 0$

Two equal real roots

3 $b^2 - 4ac < 0$



Discriminant $= (-1)^2 - 4 \times 2 \times 3$
$= -23 < 0$

No real roots

Worked example

The equation $x^2 + 4qx + 2q = 0$, where q is a non-zero constant, has equal roots.

Find the value of q. **(4 marks)**

$$b^2 - 4ac = 0$$
$$(4q)^2 - 4 \times 1 \times (2q) = 0$$
$$16q^2 - 8q = 0$$
$$q(16q - 8) = 0$$
~~$q = 0$~~ $\quad 16q - 8 = 0$ so $q = \frac{1}{2}$

Follow these steps:

1. Work out the values of a, b and c: $a = 1$, $b = 4q$ and $c = 2q$.
2. Find an expression for the discriminant ($b^2 - 4ac$) in terms of q.
3. Set the discriminant equal to 0, because there are two **equal** roots.
4. Solve this **new** equation to work out two possible values for q.

You are told that q is non-zero, so the correct solution is $q = \frac{1}{2}$.

Problem solved!

The equation must be in the form $ax^2 + bx + c = 0$ before you work out the values of a, b and c. Always write down the condition for the discriminant that you are using, and use **brackets** when you substitute.

You will need to use problem-solving skills throughout your exam – **be prepared!**

Worked example

The equation $2x^2 - kx + 6 = k$ has no real solutions for x. Show that $k^2 + 8k - 48 < 0$ **(3 marks)**

$$2x^2 - kx + 6 - k = 0$$
$$b^2 - 4ac < 0$$
$$(-k)^2 - 4 \times 2 \times (6 - k) < 0$$
$$k^2 + 8k - 48 < 0$$

Now try this

1 Find the value of the discriminant of $3x^2 - 2x - 5$ **(1 mark)**

2 The equation $px^2 + 2x - 3 = 0$, where p is a constant, has equal roots. Find the value of p. **(3 marks)**

The expression $(k + p)^2$ must always be greater than or equal to zero.

3 $f(x) = x^2 + (k + 5)x + 2k$, where k is a constant.

(a) Find the discriminant of $f(x)$ in terms of k. **(2 marks)**

(b) Show that the discriminant can be written in the form $(k + p)^2 + q$, where p and q are integers to be found. **(2 marks)**

(c) Show that, for all values of k, the equation $f(x) = 0$ has distinct real roots. **(2 marks)**

7. The discriminant

1 $(-2)^2 - 4 \times 3 \times (-5) = 64$

2 $$\begin{aligned} b^2 - 4ac &= 0 \\ 2^2 - 4p \times (-3) &= 0 \\ 4 + 12p &= 0 \\ p &= -\tfrac{1}{3} \end{aligned}$$

3 (a) $$\begin{aligned} (k+5)^2 - 4 \times 1 \times 2k &= k^2 + 10k + 25 - 8k \\ &= k^2 + 2k + 25 \end{aligned}$$

(b) $$\begin{aligned} k^2 + 2k + 25 &= (k+1)^2 - 1^2 + 25 \\ &= (k+1)^2 + 24 \end{aligned}$$

$p = 1, q = 24$

(c) $(k+1)^2 \geqslant 0$ for all k, so discriminant > 0 for all k, so $f(x) = 0$ has distinct real roots.