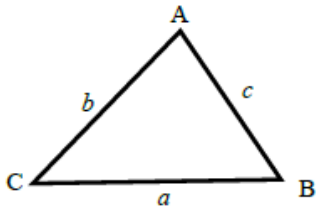
# The Cosine Rule

**A LEVEL LINKS**
**Scheme of work:** Ch3-2. Trigonometric ratios and graphs

## Key points

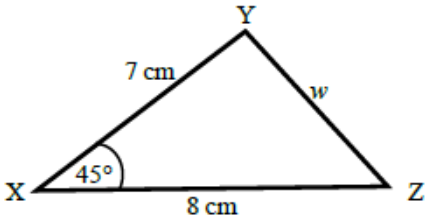
- *a* is the side opposite angle A.
  *b* is the side opposite angle B.
  *c* is the side opposite angle C.

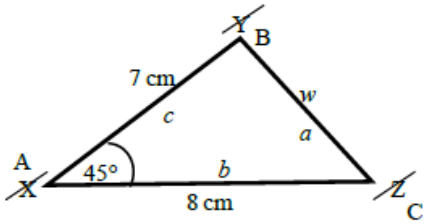



- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

## Examples

**Example 4** Work out the length of side *w*.
Give your answer correct to 3 significant figures.




$a^2 = b^2 + c^2 - 2bc\cos A$

$w^2 = 8^2 + 7^2 - 2\times 8\times 7\times \cos 45°$

$w^2 = 33.804\,040\,51...$
$w = \sqrt{33.804\,040\,51}$
$w = 5.81$ cm

**1** Always start by labelling the angles and sides.

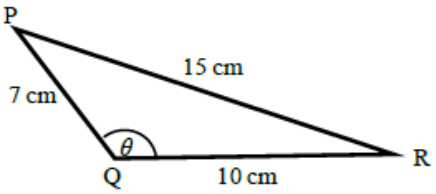
**2** Write the cosine rule to find the side.

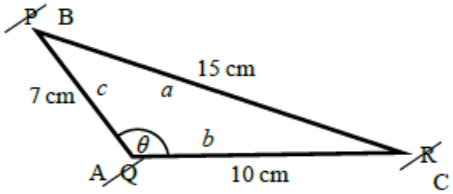
**3** Substitute the values *a*, *b* and *A* into the formula.

**4** Use a calculator to find $w^2$ and then *w*.

**5** Round your final answer to 3 significant figures and write the units in your answer.

**Example 5** Work out the size of angle $\theta$.
Give your answer correct to 1 decimal place.




$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$$

$$\cos\theta = \frac{-76}{140}$$

$\theta = 122.878\,349...$

$\theta = 122.9°$

**1** Always start by labelling the angles and sides.

**2** Write the cosine rule to find the angle.

**3** Substitute the values $a$, $b$ and $c$ into the formula.

**4** Use $\cos^{-1}$ to find the angle.

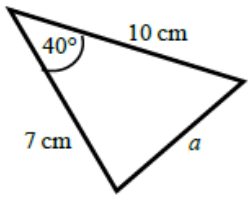
**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.

**6** Round your answer to 1 decimal place and write the units in your answer.

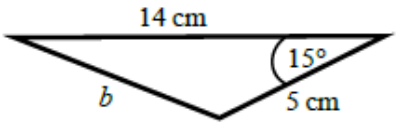
## Practice

**6** Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

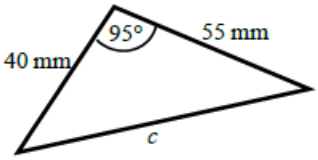
**a**



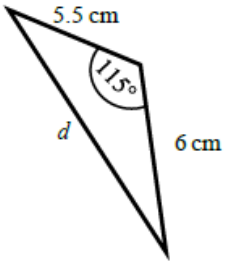

**b**




**c**

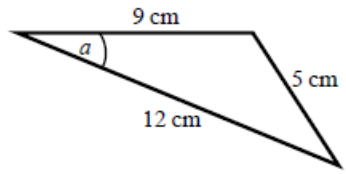



**d**

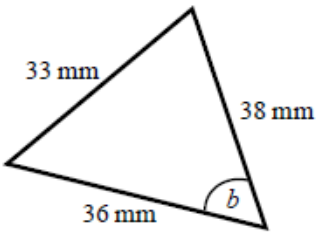


**7** Calculate the angles labelled $\theta$ in each triangle.
Give your answer correct to 1 decimal place.

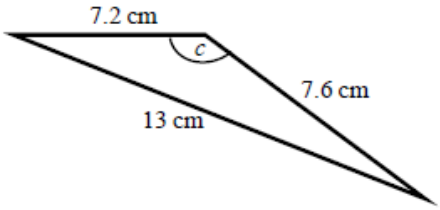
**a**



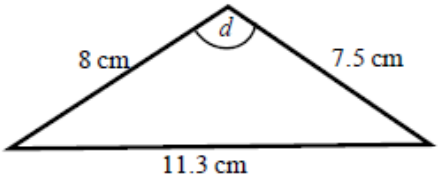

**b**




**c**

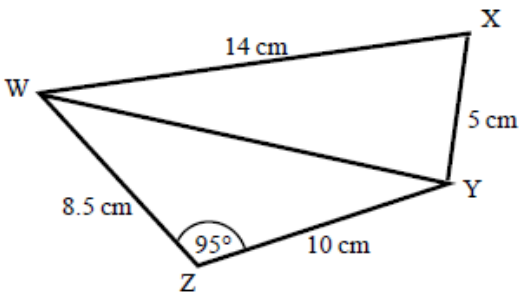



**d**




**8** **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

**b** Work out the size of angle WXY.
Give your answer correct to
1 decimal place.



# The Sine Rule

**A LEVEL LINKS**
**Scheme of work:** Ch3-2. Trigonometric ratios and graphs

## Key points

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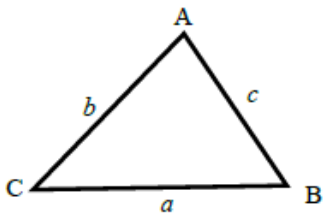
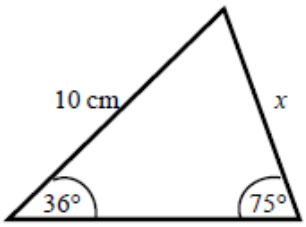
  *b* is the side opposite angle B.
  *c* is the side opposite angle C.

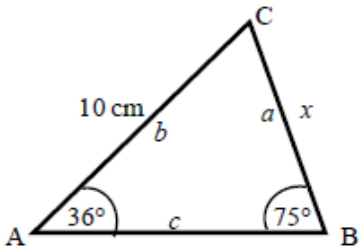



- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

## Examples

**Example 6** Work out the length of side *x*.
Give your answer correct to 3 significant figures.




$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$

$x = \frac{10 \times \sin 36°}{\sin 75°}$

$x = 6.09$ cm

**1** Always start by labelling the angles and sides.

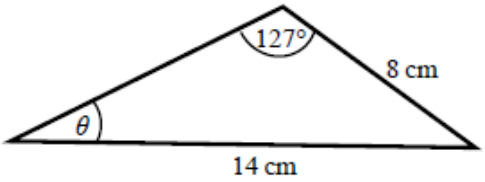
**2** Write the sine rule to find the side.

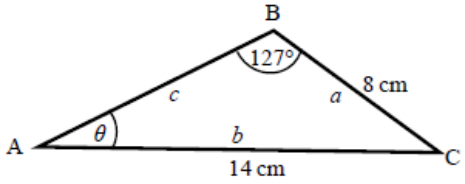
**3** Substitute the values *a*, *b*, *A* and *B* into the formula.

**4** Rearrange to make *x* the subject.

**5** Round your answer to 3 significant figures and write the units in your answer.

**Example 7** Work out the size of angle $\theta$.
Give your answer correct to 1 decimal place.




$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin\theta}{8} = \frac{\sin 127°}{14}$$

$$\sin\theta = \frac{8 \times \sin 127°}{14}$$

$$\theta = 27.2°$$

**1** Always start by labelling the angles and sides.

**2** Write the sine rule to find the angle.

**3** Substitute the values $a$, $b$, $A$ and $B$ into the formula.

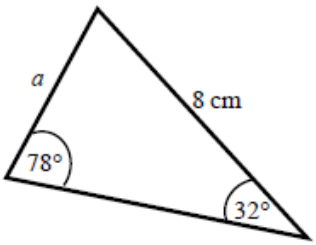
**4** Rearrange to make $\sin\theta$ the subject.

**5** Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

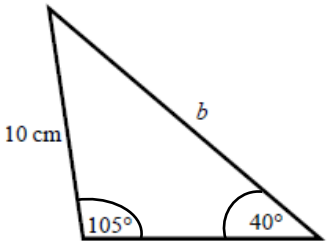
## Practice

**9** Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

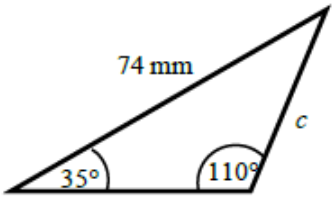
**a**



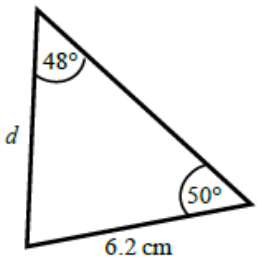

**b**




**c**

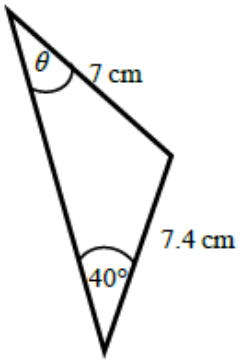



**d**

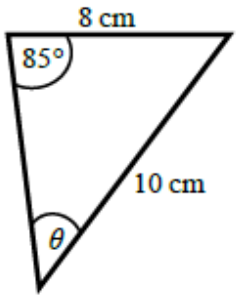


**10** Calculate the angles labelled $\theta$ in each triangle.
Give your answer correct to 1 decimal place.

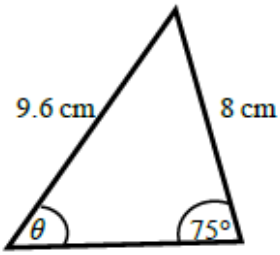
**a**



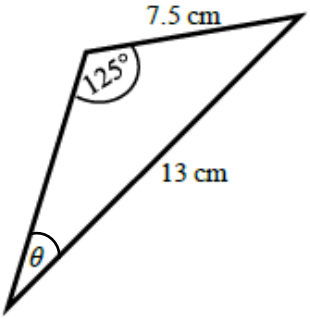

**b**




**c**

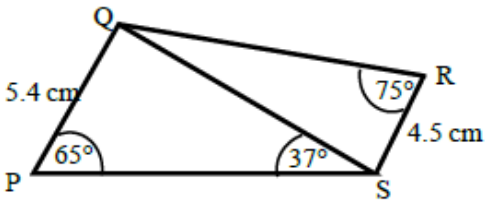



**d**




**11** **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

**b** Work out the size of angle RQS.
Give your answer correct to 1 decimal place.

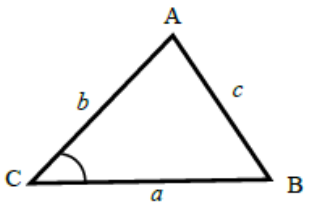


# Areas of Triangles *(Half ab SinC)*

**A LEVEL LINKS**
**Scheme of work:** Ch3-2. Trigonometric ratios and graphs

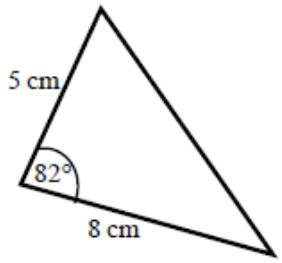
## Key points

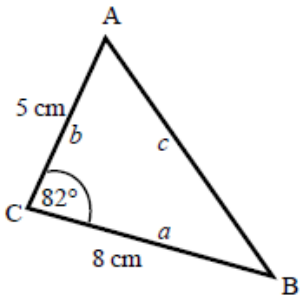
- *a* is the side opposite angle A.
  *b* is the side opposite angle B.
  *c* is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.




## Examples

**Example 8** Find the area of the triangle.




Area = $\frac{1}{2}ab\sin C$

Area = $\frac{1}{2}\times 8\times 5\times \sin 82°$

Area = 19.805 361...

Area = 19.8 $cm^2$

**1** Always start by labelling the sides and angles of the triangle.

**2** State the formula for the area of a triangle.

**3** Substitute the values of *a*, *b* and *C* into the formula for the area of a triangle.

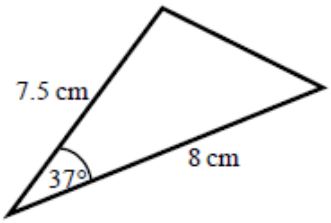
**4** Use a calculator to find the area.

**5** Round your answer to 3 significant figures and write the units in your answer.

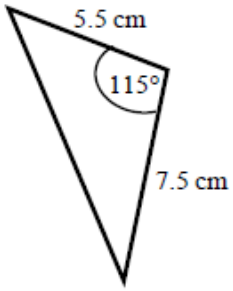
## Practice

**12** Work out the area of each triangle.
Give your answers correct to 3 significant figures.

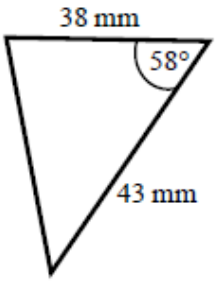
**a**



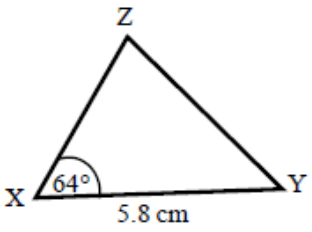

**b**




**c**




**13** The area of triangle XYZ is 13.3 $cm^2$.
Work out the length of XZ.




**Hint:**

Rearrange the formula to make a side the subject.

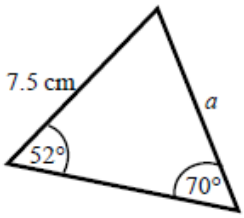
## Extend

**14** Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

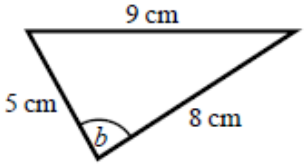
**Hint:**

For each one, decide whether to use the cosine or sine rule.

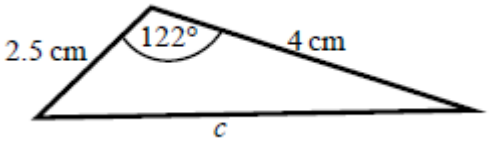
**a**



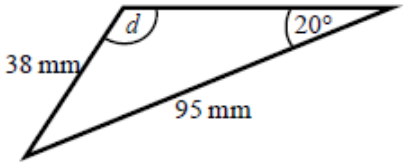

**b**



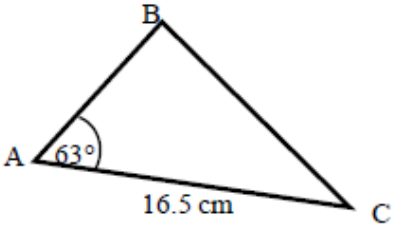
**c**




**d**




**15** The area of triangle ABC is 86.7 $cm^2$.
Work out the length of BC.
Give your answer correct to 3 significant figures.



# Answers

**6** **a** 6.46 cm **b** 9.26 cm **c** 70.8 mm **d** 9.70 cm

**7** **a** 22.2° **b** 52.9° **c** 122.9° **d** 93.6°

**8** **a** 13.7 cm **b** 76.0°

**9** **a** 4.33 cm **b** 15.0 cm **c** 45.2 mm **d** 6.39 cm

**10** **a** 42.8° **b** 52.8° **c** 53.6° **d** 28.2°

**11** **a** 8.13 cm **b** 32.3°

**12** **a** 18.1 $cm^2$ **b** 18.7 $cm^2$ **c** 693 $mm^2$

**13** 5.10 cm

**14** **a** 6.29 cm **b** 84.3° **c** 5.73 cm **d** 58.8°

**15** 15.3 cm