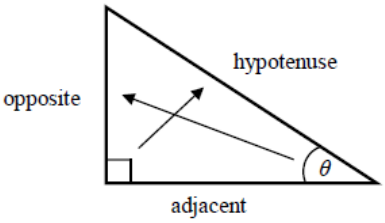
Trigonometry in Right-Angled Triangles

A LEVEL LINKS
Scheme of work: Ch3-1. Trigonometric ratios and graphs

Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.



- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

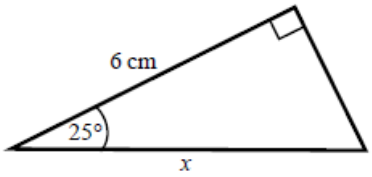
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

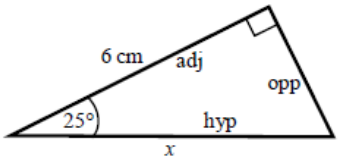
- The sine, cosine and tangent of some angles may be written exactly.

	0	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

Examples

Example 1 Calculate the length of side x.
Give your answer correct to 3 significant figures.





$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$\cos 25° = \dfrac{6}{x}$

$x = \dfrac{6}{\cos 25°}$

$x = 6.620\,267\,5...$

$x = 6.62$ cm

1 Always start by labelling the sides.

2 You are given the adjacent and the hypotenuse so use the cosine ratio.

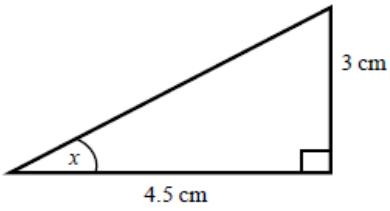
3 Substitute the sides and angle into the cosine ratio.

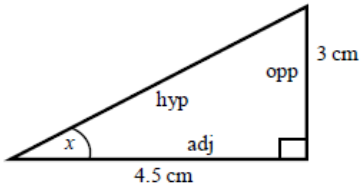
4 Rearrange to make x the subject.

5 Use your calculator to work out $6 \div \cos 25°$.

6 Round your answer to 3 significant figures and write the units in your answer.

Example 2 Calculate the size of angle x.
Give your answer correct to 3 significant figures.





$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan x = \dfrac{3}{4.5}$

$x = \tan^{-1}\left(\dfrac{3}{4.5}\right)$

$x = 33.690\,067\,5...$

$x = 33.7°$

1 Always start by labelling the sides.

2 You are given the opposite and the adjacent so use the tangent ratio.

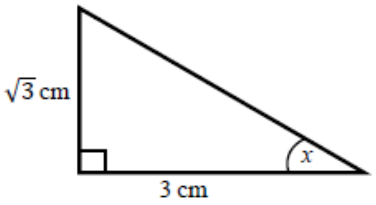
3 Substitute the sides and angle into the tangent ratio.

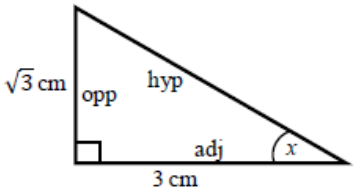
4 Use $\tan^{-1}$ to find the angle.

5 Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.

6 Round your answer to 3 significant figures and write the units in your answer.

Example 3 Calculate the exact size of angle x.





$\tan\theta = \frac{\text{opp}}{\text{adj}}$

$\tan x = \frac{\sqrt{3}}{3}$

$x = 30°$

1 Always start by labelling the sides.

2 You are given the opposite and the adjacent so use the tangent ratio.

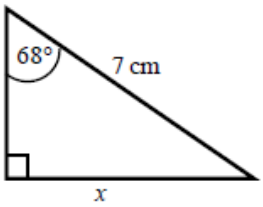
3 Substitute the sides and angle into the tangent ratio.

4 Use the table from the key points to find the angle.

Practice

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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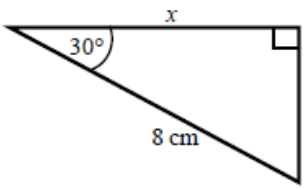

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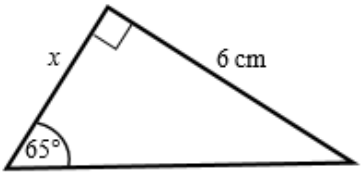

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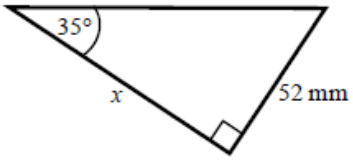

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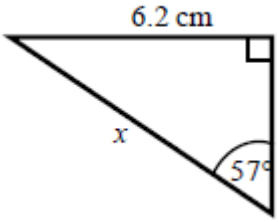

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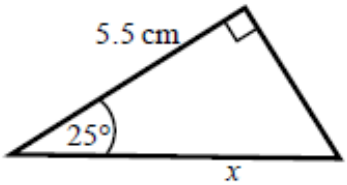
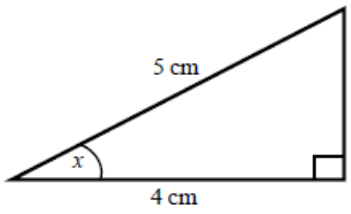


2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.

a



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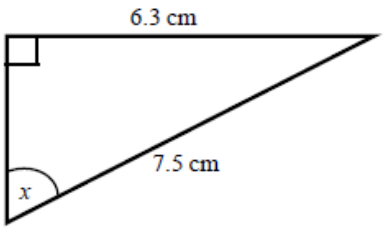

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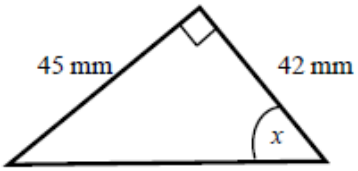

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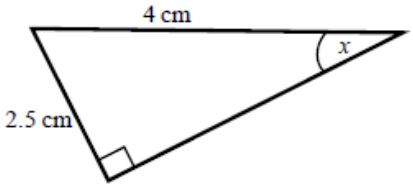
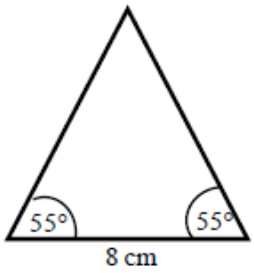


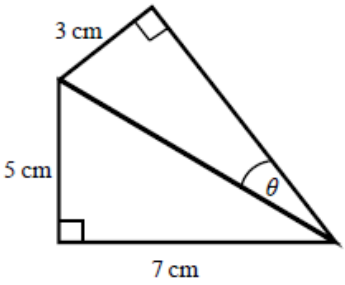
3 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

> **Hint:**
>
> Split the triangle into two right-angled triangles.



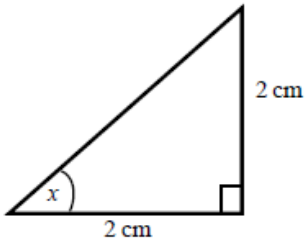
4 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

> **Hint:**
>
> First work out the length of the common side to both triangles, leaving your answer in surd form.



5 Find the exact value of x in each triangle.

a



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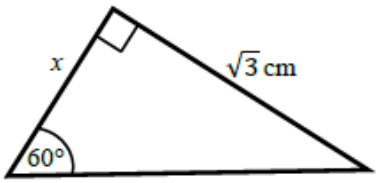

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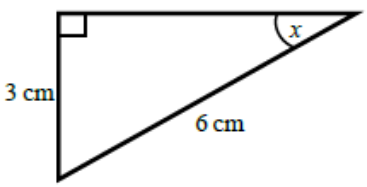

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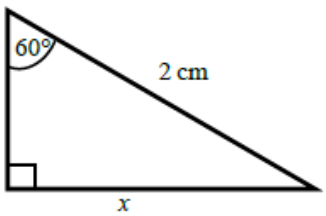

Answers

1 **a** 6.49 cm **b** 6.93 cm **c** 2.80 cm
d 74.3 mm **e** 7.39 cm **f** 6.07 cm

2 **a** 36.9° **b** 57.1° **c** 47.0° **d** 38.7°

3 5.71 cm

4 20.4°

5 **a** 45° **b** 1 cm **c** 30° **d** $\sqrt{3}$ cm