

# Rearranging Equations

## A LEVEL LINKS

**Scheme of work:** Ch2-5. Definition, differentiating polynomials, second derivatives

## Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

## Examples

**Example 1** Make  $t$  the subject of the formula  $v = u + at$ .

$v = u + at$ $v - u = at$ $t = \frac{v - u}{a}$	<ol style="list-style-type: none"> <li>1 Get the terms containing <math>t</math> on one side and everything else on the other side.</li> <li>2 Divide throughout by <math>a</math>.</li> </ol>
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**Example 2** Make  $t$  the subject of the formula  $r = 2t - \pi t$ .

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2 - \pi}$	<ol style="list-style-type: none"> <li>1 All the terms containing <math>t</math> are already on one side and everything else is on the other side.</li> <li>2 Factorise as <math>t</math> is a common factor.</li> <li>3 Divide throughout by <math>2 - \pi</math>.</li> </ol>
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**Example 3** Make  $t$  the subject of the formula  $\frac{t+r}{5} = \frac{3t}{2}$ .

$\frac{t+r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$	<ol style="list-style-type: none"> <li>1 Remove the fractions first by multiplying throughout by 10.</li> <li>2 Get the terms containing <math>t</math> on one side and everything else on the other side and simplify.</li> <li>3 Divide throughout by 13.</li> </ol>
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**Example 4** Make  $t$  the subject of the formula  $r = \frac{3t+5}{t-1}$ .

$r = \frac{3t+5}{t-1}$ $r(t-1) = 3t+5$ $rt - r = 3t+5$ $rt - 3t = 5+r$ $t(r-3) = 5+r$ $t = \frac{5+r}{r-3}$	<ol style="list-style-type: none"> <li>1 Remove the fraction first by multiplying throughout by <math>t-1</math>.</li> <li>2 Expand the brackets.</li> <li>3 Get the terms containing <math>t</math> on one side and everything else on the other side.</li> <li>4 Factorise the LHS as <math>t</math> is a common factor.</li> <li>5 Divide throughout by <math>r-3</math>.</li> </ol>
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## Practice

Change the subject of each formula to the letter given in the brackets.

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|---|-----------------------------------|-----------------------------------|
| 1 $C = \pi d$ [ $d$ ]                       | 2 $P = 2l + 2w$ [ $w$ ]           | 3 $D = \frac{S}{T}$ [ $T$ ]       |
| 4 $p = \frac{q-r}{t}$ [ $t$ ]               | 5 $u = at - \frac{1}{2}t$ [ $t$ ] | 6 $V = ax + 4x$ [ $x$ ]           |
| 7 $\frac{y-7x}{2} = \frac{7-2y}{3}$ [ $y$ ] | 8 $x = \frac{2a-1}{3-a}$ [ $a$ ]  | 9 $x = \frac{b-c}{d}$ [ $d$ ]     |
| 10 $h = \frac{7g-9}{2+g}$ [ $g$ ]           | 11 $e(9+x) = 2e+1$ [ $e$ ]        | 12 $y = \frac{2x+3}{4-x}$ [ $x$ ] |

13 Make  $r$  the subject of the following formulae.

a $A = \pi r^2$	b $V = \frac{4}{3}\pi r^3$	c $P = \pi r + 2r$	d $V = \frac{2}{3}\pi r^2 h$
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14 Make  $x$  the subject of the following formulae.

a $\frac{xy}{z} = \frac{ab}{cd}$	b $\frac{4\pi cx}{d} = \frac{3z}{py^2}$
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15 Make  $\sin B$  the subject of the formula  $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make  $\cos B$  the subject of the formula  $b^2 = a^2 + c^2 - 2ac \cos B$ .

## Extend

17 Make  $x$  the subject of the following equations.

a $\frac{p}{q}(sx+t) = x-1$	b $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$
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## Answers

$$1 \quad d = \frac{C}{\pi}$$

$$2 \quad w = \frac{P-2l}{2}$$

$$3 \quad T = \frac{S}{D}$$

$$4 \quad t = \frac{q-r}{p}$$

$$5 \quad t = \frac{2u}{2a-1}$$

$$6 \quad x = \frac{V}{a+4}$$

$$7 \quad y = 2 + 3x$$

$$8 \quad a = \frac{3x+1}{x+2}$$

$$9 \quad d = \frac{b-c}{x}$$

$$10 \quad g = \frac{2h+9}{7-h}$$

$$11 \quad e = \frac{1}{x+7}$$

$$12 \quad x = \frac{4y-3}{2+y}$$

$$13 \quad \mathbf{a} \quad r = \sqrt{\frac{A}{\pi}}$$

$$\mathbf{b} \quad r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$\mathbf{c} \quad r = \frac{P}{\pi+2}$$

$$\mathbf{d} \quad r = \sqrt{\frac{3V}{2\pi h}}$$

$$14 \quad \mathbf{a} \quad x = \frac{abz}{cdy}$$

$$\mathbf{b} \quad x = \frac{3dz}{4\pi cpy^2}$$

$$15 \quad \sin B = \frac{b \sin A}{a}$$

$$16 \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$17 \quad \mathbf{a} \quad x = \frac{q+pt}{q-ps}$$

$$\mathbf{b} \quad x = \frac{3py+2pqy}{3p-apq} = \frac{y(3+2q)}{3-aq}$$