

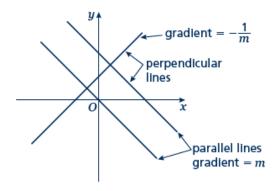
Parallel and Perpendicular Lines

A LEVEL LINKS

Scheme of work: Ch1-5. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation y = mx + c has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to y = 2x + 4 which passes through the point (4, 9).

$$y = 2x + 4$$

 $m = 2$
 $y = 2x + c$ 1 As the lines are parallel they have
the same gradient. $9 = 2 \times 4 + c$ 2 Substitute $m = 2$ into the equation of
a straight line $y = mx + c$. $9 = 8 + c$
 $c = 1$
 $y = 2x + 1$ 3 Substitute the coordinates into the
equation $y = 2x + c$ 4 Simplify and solve the equation.5 Substitute $c = 1$ into the equation
 $y = 2x + c$

Example 2 Find the equation of the line perpendicular to y = 2x - 3 which passes through the point (-2, 5).

$$y = 2x - 3$$

$$m = 2$$

$$-\frac{1}{m} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x + c$$

$$5 = -\frac{1}{2} \times (-2) + c$$

$$5 = 1 + c$$

$$c = 4$$

$$y = -\frac{1}{2}x + 4$$

1 As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.

2 Substitute $m = -\frac{1}{2}$ into $y = mx + c$.

3 Substitute the coordinates $(-2, 5)$ into the equation $y = -\frac{1}{2}x + c$

4 Simplify and solve the equation.

5 Substitute $c = 4$ into $c = 4$ into





Example 3 A line passes through the points (0, 5) and (9, -1). Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$$x_1 = 0$$
, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{9 - 0}$$

$$= \frac{-6}{9} = -\frac{2}{3}$$
1 3

Midpoint =
$$\left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$$

$$2 = \frac{3}{2} \times \frac{9}{2} + \epsilon$$

$$c = -\frac{19}{4}$$

$$y = \frac{3}{2}x - \frac{19}{4}$$

- 1 Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
- 2 As the lines are perpendicular, the gradient of the perpendicular line
- 3 Substitute the gradient into the equation y = mx + c.
- 4 Work out the coordinates of the midpoint of the line.
- 5 Substitute the coordinates of the midpoint into the equation.
- **6** Simplify and solve the equation.
- 7 Substitute $c = -\frac{19}{4}$ into the equation

$$y = \frac{3}{2}x + c .$$

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a
$$y = 3x + 1$$
 (3, 2)

b
$$y = 3 - 2x$$
 (1, 3)

$$\mathbf{c} \qquad 2x + 4y + 3 = 0 \quad (6, -3)$$

$$y = 3x + 1$$
 (3, 2)
 $2x + 4y + 3 = 0$ (6, -3)
 b $y = 3 - 2x$ (1, 3)
 d $2y - 3x + 2 = 0$ (8, 20)

Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which 2 passes through the point (-5, 3).

Fint
$$f m = \frac{a}{b}$$
 then the neg

If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

Find the equation of the line perpendicular to each of the given lines and which passes through 3 each of the given points.

a
$$y = 2x - 6$$
 (4, 0)

b
$$y = -\frac{1}{3}x + \frac{1}{2}$$
 (2, 13)

c
$$x-4y-4=0$$
 (5, 15) **d** $5y+2x-5=0$ (6, 7)

$$\mathbf{d} \quad 5y + 2x - 5 = 0 \quad (6, 7)$$



4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

$$a$$
 (4, 3), (-2, -9)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

$$\mathbf{a} \qquad y = 2x + 3$$
$$y = 2x - 7$$

$$y = 3x$$

$$2x + y - 3 = 0$$

$$\mathbf{c} \qquad y = 4x - 3$$
$$4y + x = 2$$

$$\mathbf{d} \qquad 3x - y + 5 = 0$$
$$x + 3y = 1$$

e
$$2x + 5y - 1 = 0$$
 f $y = 2x + 7$

$$\mathbf{f} \qquad 2x - y = 6$$
$$6x - 3y + 3 = 0$$

- 6 The straight line L_1 passes through the points A and B with coordinates (-4, 4) and (2, 1), respectively.
 - a Find the equation of L_1 in the form ax + by + c = 0

The line L_2 is parallel to the line L_1 and passes through the point C with coordinates (-8, 3).

b Find the equation of \mathbf{L}_2 in the form ax + by + c = 0

The line L_3 is perpendicular to the line L_1 and passes through the origin.

c Find an equation of L₃



Answers

1 **a**
$$y = 3x - 7$$

$$\mathbf{b} \qquad y = -2x + 5$$

c
$$y = -\frac{1}{2}x$$

1 **a**
$$y = 3x - 7$$
 b $y = -2x + 5$
c $y = -\frac{1}{2}x$ **d** $y = \frac{3}{2}x + 8$

2
$$y = -2x - 7$$

3 a
$$y = -\frac{1}{2}x + 2$$
 b $y = 3x + 7$

$$\mathbf{b} \qquad y = 3x + 7$$

c
$$y = -4x + 35$$

c
$$y = -4x + 35$$
 d $y = \frac{5}{2}x - 8$

4 a
$$y = -\frac{1}{2}x$$

$$\mathbf{b} \qquad y = 2x$$

b Neither

Perpendicular

d Perpendicular

e Neither

Parallel

6 a
$$x + 2y - 4 = 0$$
 b $x + 2y + 2 = 0$ **c** $y = 2x$

b
$$x + 2y + 2 = 0$$

$$\mathbf{c}$$
 $y = 2x$