Rules of Indices

> **A LEVEL LINKS**
> **Scheme of work:** Ch1-2. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1   Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2   Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $\quad = 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3   Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $\quad = 3^2$ $\quad = 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4   Evaluate 4^{-2}

$4^{-2} = \dfrac{1}{4^2}$ $= \dfrac{1}{16}$	**1** Use the rule $a^{-m} = \dfrac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5   Simplify $\dfrac{6x^5}{2x^2}$

$\dfrac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\dfrac{a^m}{a^n} = a^{m-n}$ to give $\dfrac{x^5}{x^2} = x^{5-2} = x^3$

Example 6   Simplify $\dfrac{x^3 \times x^5}{x^4}$

$\dfrac{x^3 \times x^5}{x^4} = \dfrac{x^{3+5}}{x^4} = \dfrac{x^8}{x^4}$ $\qquad = x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\dfrac{a^m}{a^n} = a^{m-n}$

Example 7   Write $\dfrac{1}{3x}$ as a single power of x

$\dfrac{1}{3x} = \dfrac{1}{3}x^{-1}$	Use the rule $\dfrac{1}{a^m} = a^{-m}$, note that the fraction $\dfrac{1}{3}$ remains unchanged

Example 8   Write $\dfrac{4}{\sqrt{x}}$ as a single power of x

$\dfrac{4}{\sqrt{x}} = \dfrac{4}{x^{\frac{1}{2}}}$ $\qquad = 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\dfrac{1}{a^m} = a^{-m}$

Practice

1 Evaluate.

  a 14^0 **b** 3^0 **c** 5^0 **d** x^0

2 Evaluate.

  a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

3 Evaluate.

  a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

4 Evaluate.

  a 5^{-2} **b** 4^{-3} **c** 2^{-5} **d** 6^{-2}

5 Simplify.

  a $\dfrac{3x^2 \times x^3}{2x^2}$ **b** $\dfrac{10x^5}{2x^2 \times x}$

  c $\dfrac{3x \times 2x^3}{2x^3}$ **d** $\dfrac{7x^3 y^2}{14x^5 y}$

  e $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

  g $\dfrac{\left(2x^2\right)^3}{4x^0}$ **h** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

> **Watch out!**
>
> Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

  a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

  d $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\dfrac{9}{16}\right)^{-\frac{1}{2}}$ **f** $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x.

  a $\dfrac{1}{x}$ **b** $\dfrac{1}{x^7}$ **c** $\sqrt[4]{x}$

  d $\sqrt[5]{x^2}$ **e** $\dfrac{1}{\sqrt[3]{x}}$ **f** $\dfrac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

    a x^{-3} **b** x^0 **c** $x^{\frac{1}{5}}$

    d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{2}}$ **f** $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n.

    a $5\sqrt{x}$ **b** $\dfrac{2}{x^3}$ **c** $\dfrac{1}{3x^4}$

    d $\dfrac{2}{\sqrt{x}}$ **e** $\dfrac{4}{\sqrt[3]{x}}$ **f** 3

Extend

10 Write as sums of powers of x.

    a $\dfrac{x^5+1}{x^2}$ **b** $x^2\left(x+\dfrac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\dfrac{1}{x^3}\right)$

Answers

1 **a** 1   **b** 1   **c** 1   **d** 1

2 **a** 7   **b** 4   **c** 5   **d** 2

3 **a** 125   **b** 32   **c** 343   **d** 8

4 **a** $\dfrac{1}{25}$ **b** $\dfrac{1}{64}$ **c** $\dfrac{1}{32}$ **d** $\dfrac{1}{36}$

5 **a** $\dfrac{3x^3}{2}$ **b** $5x^2$

      c $3x$ **d** $\dfrac{y}{2x^2}$

      e $y^{\frac{1}{2}}$ **f** c^{-3}
      g $2x^6$ **h** x

6 **a** $\dfrac{1}{2}$ **b** $\dfrac{1}{9}$ **c** $\dfrac{8}{3}$

      d $\dfrac{1}{4}$ **e** $\dfrac{4}{3}$ **f** $\dfrac{16}{9}$

7 **a** x^{-1} **b** x^{-7} **c** $x^{\frac{1}{4}}$

      d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{3}}$ **f** $x^{-\frac{2}{3}}$

8 **a** $\dfrac{1}{x^3}$ **b** 1   **c** $\sqrt[5]{x}$

      d $\sqrt[5]{x^2}$ **e** $\dfrac{1}{\sqrt{x}}$ **f** $\dfrac{1}{\sqrt[4]{x^3}}$

9 **a** $5x^{\frac{1}{2}}$ **b** $2x^{-3}$ **c** $\dfrac{1}{3}x^{-4}$

      d $2x^{-\frac{1}{2}}$ **e** $4x^{-\frac{1}{3}}$ **f** $3x^0$

10 **a** $x^3 + x^{-2}$ **b** $x^3 + x$ **c** $x^{-2} + x^{-7}$