



AQA Qualifications

AQA Level 2 Certificate

FURTHER MATHEMATICS

Level 2 (8360)

Worksheet 3
Algebraic Proof

Our specification is published on our website (www.aqa.org.uk). We will let centres know in writing about any changes to the specification. We will also publish changes on our website. The definitive version of our specification will always be the one on our website, this may differ from printed versions.

You can get further copies of this Teacher Resource from:

The GCSE Mathematics Department

AQA

Devas Street

Manchester

M16 6EX

Or, you can download a copy from our All About Maths website (<http://allaboutmaths.aqa.org.uk/>).

Copyright © 2012 AQA and its licensors. All rights reserved.

AQA retains the copyright on all its publications, including the specifications. However, registered centres for AQA are permitted to copy material from this specification booklet for their own internal use.

AQA Education (AQA) is a registered charity (number 1073334) and a company limited by guarantee registered in England and Wales (number 3644723). Our registered address is AQA, Devas Street, Manchester M15 6EX.

3 Algebraic Proof

Question 1

Prove that $4(p - 3) - 2(2p - 1)$ is always a negative integer. (2 marks)

Question 2

Prove that $8(y + 3) + 3(2 - y)$ is a multiple of 5 when y is a positive integer. (3 marks)

Question 3

a is a positive integer.

Prove that $4a^2(2a + 1) - (2a)^2$ is a cube number. (3 marks)

Question 4

a and b are positive integers.

$a < b$

Prove that $\frac{ax + 3a}{bx + 3b} < 1$ $x \neq -3$ (3 marks)

Question 5

(a) Express $x^2 + 6x + 11$ in the form $(x + a)^2 + b$ where a and b are integers. (2 marks)

(b) Hence, prove that $x^2 + 6x + 11$ is always positive. (2 marks)

Question 6

Prove that, for all values of x , $x^2 + 2x + 6 > 0$ (4 marks)

Question 7

$f(x) = (2x + 3)^2 + 8(x + 2)$ for all values of x .

Prove that there is exactly one value of x for which $f(x) = 0$ (4 marks)

Question 8

The n th term of a sequence is $\frac{1}{2}n(n+1)$

- (a) Work out an expression for the $(n-1)$ th term of the sequence.
Give your answer in its simplest form. *(2 marks)*
- (b) Hence, or otherwise, prove that the sum of any consecutive pair of terms of the sequence is a square number. *(3 marks)*

Question 9

Prove that $\frac{x^2-4}{5x-10} \times \frac{10x^2}{x+2}$ is always positive. *(5 marks)*

Question 10

$$f(n) = n^2 - n$$

Prove that $f(3n) + f(n+1) = kn(5n-1)$ where k is an integer. *(3 marks)*