AQA Qualifications

AQA Level 2 Certificate FURTHER MATHEMATICS

Level 2 (8360)

Worksheet 3

Algebraic Proof

Version 2.0

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The GCSE Mathematics Department
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3 Algebraic Proof

Question 1

Prove that $4(p - 3) - 2(2p - 1)$ is always a negative integer. *(2 marks)*

Question 2

Prove that $8(y + 3) + 3(2 - y)$ is a multiple of 5 when y is a positive integer. *(3 marks)*

Question 3

a is a positive integer.

Prove that $4a^2(2a + 1) - (2a)^2$ is a cube number. *(3 marks)*

Question 4

a and b are positive integers.

$a < b$

Prove that $\frac{ax + 3a}{bx + 3b} < 1 \qquad x \neq -3$ *(3 marks)*

Question 5

(a) Express $x^2 + 6x + 11$ in the form $(x + a)^2 + b$ where a and b are integers. *(2 marks)*

(b) Hence, prove that $x^2 + 6x + 11$ is always positive. *(2 marks)*

Question 6

Prove that, for all values of x, $x^2 + 2x + 6 > 0$ *(4 marks)*

Question 7

$f(x) = (2x + 3)^2 + 8(x + 2)$ for all values of x.

Prove that there is exactly one value of x for which $f(x) = 0$ *(4 marks)*

Question 8

The nth term of a sequence is $\frac{1}{2}n(n+1)$

(a) Work out an expression for the $(n-1)$th term of the sequence.

Give your answer in its simplest form. *(2 marks)*

(b) Hence, or otherwise, prove that the sum of any consecutive pair of terms of the sequence is a square number. *(3 marks)*

Question 9

Prove that $\frac{x^2-4}{5x-10} \times \frac{10x^2}{x+2}$ is always positive. *(5 marks)*

Question 10

$f(n) = n^2 - n$

Prove that $f(3n) + f(n+1) = kn(5n-1)$ where k is an integer. *(3 marks)*